

# CONFORMALLY FLAT SPHERICALLY SYMMETRIC COSMOLOGICAL MODELS-REVISITED

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## Abstract

Conformally flat spherically symmetric cosmological models representing a charged perfect fluid as well as a bulk viscous fluid distribution have been obtained. The cosmological constant  $\Lambda$  is found positive and is a decreasing function of time which is consistent with the recent supernovae observations. The physical and geometrical properties of the models are discussed.

Key Words : Cosmology; Conformally Flat Universe; Spherically Symmetric Models.

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# 1 Introduction

A considerable interest has been shown to the study of physical properties of spacetimes which are conformal to certain well known gravitational fields. The general theory of relativity is believed by a number of unknown functions - the ten components of  $g^{ij}$ . Hence there is a little hope of finding physically interesting results without making reduction in their number. In conformally flat spacetime the number of unknown functions is reduced to one. The conformally flat metrics are of particular interest in view of their degeneracy in the context of Petrov classification. A number of conformally flat physically significant spacetimes are known like Schwarzschild interior solution and Lemaître cosmological universe.

At the present state of evolution, the universe is spherically symmetric and the matter distribution in it is isotropic and homogeneous. Buchdahl [1] has obtained the conformal flatness of the Schwarzschild interior solution. Singh and Roy [2] have discussed the possibility of existence of electromagnetic fields conformal to some empty spacetime. Singh and Abdussattar [3] have obtained a non-static generalization of Schwarzschild interior solution which is conformal to flat spacetime. Roy and Bali [4] have obtained the solution of Einstein's field equations representing non-static spherically symmetric perfect fluid distribution which is conformally flat. Pandey and Tiwari [5] have discussed conformally flat spherically symmetric charged perfect fluid distribution. Reddy [6] and Rao and Reddy [7] discussed static conformally flat solutions in the Brans-Dicke and Nordtvedt-Barker scalar-tensor theories. Shanthi [8] has shown that the most general conformally flat static vacuum solution in the Nordtvedt-Barker scalar-tensor theory is simply the empty flat spacetime of general relativity. There has been a recent literature (Melfo and Rago [9], Mannheim [10], Yadav and Prasad [11], Endean [12,13], Obukhov et al. [14], Mark and Harko [15]) which shows a significant interest in the study of conformally flat spacetime.

Most cosmological models assume that the matter in the universe can be described by 'dust' (a pressureless distribution) or at best a perfect fluid. Nevertheless, there is good reason to believe that - at least at the early stages of the universe - viscous effects do play a role [17]-[19]. For example, the existence of the bulk viscosity is equivalent to slow process of restoring equilibrium states [20]. The observed physical phenomena such as the large entropy

per baryon and remarkable degree of isotropy of the cosmic microwave background radiation suggest analysis of dissipative effects in cosmology. Bulk viscosity is associated with the GUT phase transition and string creation. Thus, we should consider the presence of a material distribution other than a perfect fluid to have realistic cosmological models (see Grøn [32]) for a review on cosmological models with bulk viscosity). The effect of bulk viscosity on the cosmological evolution has been investigated by a number of authors in the framework of general theory of relativity [21]-[31].

The purpose of this paper is to apply conformally flat spherically symmetric line element to charged perfect fluid and to bulk viscous fluid models in cosmology. This paper is organized as follows. The field equations are presented in Section 2. Section 3 includes the solution of the field equations in presence of charged perfect fluid distribution. Section 3.1 contains some physical properties of the model. Finally in Section 4 the bulk viscous models are considered.

## 2 Field Equations

We consider the conformal metric in spherical polar coordinates

$$ds^2 = e^\lambda(dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\phi^2 - dt^2), \quad (1)$$

where  $\lambda$  is a function of  $r$  and  $t$ . We number the coordinates as  $x^1 = r$ ,  $x^2 = \theta$ ,  $x^3 = \phi$  and  $x^4 = t$ .

The energy momentum tensor for distribution of a charged perfect fluid has the form

$$T_{ij} = (\epsilon + p)v_iv_j + pg_{ij} + E_{ij}, \quad (2)$$

where  $E_{ij}$  is the electromagnetic field given by

$$E_{ij} = \frac{1}{4\pi} \left[ F_{ai}F_{bj}g^{ab} - \frac{1}{2}g_{ij}F_{ab}F^{ab} \right]. \quad (3)$$

Here  $\epsilon$  and  $p$  are the energy density and isotropic pressure respectively and  $v^i$  is the flow vector satisfying the relation

$$g_{ij}v^iv^j = -1. \quad (4)$$

The electromagnetic field tensor  $F_{ij}$  satisfies Maxwell's equations

$$F_{;j}^{ij} = 4\pi\rho v^i, \quad (5)$$

$$F_{[ij;k]} = 0, \quad (6)$$

$\rho$  being the current density. Here and henceforth a comma and a semicolon denote ordinary and covariant differentiation respectively. The Einstein field equations

$$R_{ij} - \frac{1}{2}g_{ij}R + \Lambda g_{ij} = -8\pi T_{ij}, \quad (7)$$

for the line element (1) has been set up as

$$8\pi[(\epsilon + p)v_1^2 + pe^\lambda] = \frac{3\lambda_1^2}{4} + \frac{2\lambda_1}{r} - \lambda_{44} - \frac{\lambda_4^2}{4} + e^{-\lambda}(F_{14})^2 + \Lambda e^\lambda, \quad (8)$$

$$8\pi pe^\lambda = \lambda_{11} + \frac{\lambda_1^2}{4} + \frac{\lambda_1}{r} - \lambda_{44} - \frac{\lambda_4^2}{4} - e^{-\lambda}(F_{14})^2 + \Lambda e^\lambda, \quad (9)$$

$$8\pi[(\epsilon + p)v_4^2 - pe^\lambda] = \frac{3\lambda_4^2}{4} - \lambda_{11} - \frac{\lambda_1^2}{4} - \frac{2\lambda_1}{r} - e^{-\lambda}(F_{14})^2 - \Lambda e^\lambda, \quad (10)$$

$$8\pi(\epsilon + p)v_1v_4 = \frac{\lambda_1\lambda_4}{2} - \lambda_{14}. \quad (11)$$

Equation (4) gives

$$v_4^2 - v_1^2 = e^\lambda. \quad (12)$$

### 3 Solutions of the field equations

From eqs. (8) and (9) we have

$$8\pi[(\epsilon + p)v_1^2] - 2e^{-\lambda}(F_{14})^2 = \frac{\lambda_1^2}{2} + \frac{\lambda_1}{r} - \lambda_{11}. \quad (13)$$

Also eqs. (9) and (10) readily give

$$8\pi[(\epsilon + p)v_4^2] + 2e^{-\lambda}(F_{14})^2 = \frac{\lambda_4^2}{2} - \frac{\lambda_1}{r} - \lambda_{44}. \quad (14)$$

Combining eqs. (12), (13) and (14) we obtain

$$8\pi[(\epsilon + p)e^\lambda] + 4e^{-\lambda}(F_{14})^2 = \frac{\lambda_4^2}{2} - \frac{\lambda_1^2}{2} - \frac{2\lambda_1}{r} - \lambda_{44} + \lambda_{11}. \quad (15)$$

Equations (9) and (15) together reduce to

$$8\pi\epsilon e^\lambda + 3e^{-\lambda}(F_{14})^2 = \frac{3}{4} \left( \lambda_4^2 - \lambda_1^2 - \frac{4\lambda_1}{r} \right). \quad (16)$$

In comoving coordinate system  $v_1 = 0$ , then eq. (13) reduces to

$$-e^{-\lambda}(F_{14})^2 = \frac{\lambda_1^2}{4} + \frac{\lambda_1}{2r} - \frac{\lambda_{11}}{2}. \quad (17)$$

From eq.(11) we obtain

$$2\lambda_{14} - \lambda_1\lambda_4 = 0. \quad (18)$$

The general solution of (18) is obtained as

$$e^\lambda = [\alpha(r) + \beta(t)]^{-2}, \quad (19)$$

where  $\alpha$  and  $\beta$  are functions of  $r$  and  $t$  respectively.

Hence the geometry of the spacetime (1) reduces to the form

$$ds^2 = \frac{1}{(\alpha + \beta)^2} (dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 - dt^2), \quad (20)$$

which is the model for a distribution of charged perfect fluid with the flow vector in  $t$ -direction. The pressure and density for the model (20) are given by

$$8\pi p - \Lambda = 3(\alpha_1^2 - \beta_4^2) + (\alpha + \beta) \left( 2\beta_{44} - \alpha_{11} - \frac{3\alpha_1}{r} \right), \quad (21)$$

$$8\pi\epsilon + \Lambda = 3(\beta_4^2 - \alpha_1^2) + 3(\alpha + \beta) \left( \alpha_{11} + \frac{\alpha_1}{r} \right). \quad (22)$$

Let us assume that the fluid obeys an equation of state of the form

$$p = \gamma\epsilon, \quad (23)$$

where  $\gamma(0 \leq \gamma \leq 1)$  is constant. Using eq. (23) in eqs. (21) and eq. (22), we get

$$\epsilon = \frac{(\alpha + \beta)}{8\pi(1 + \gamma)}(\beta_{44} + \alpha_{11}), \quad (24)$$

$$\begin{aligned} \Lambda = & -\frac{(1 - \gamma)}{(1 + \gamma)}(\alpha + \beta)(\beta_{44} + \alpha_{11}) - 3(\alpha_1^2 - \beta_4^2) \\ & + (\alpha + \beta)(\beta_{44} - 2\alpha_{11}). \end{aligned} \quad (25)$$

If we put  $\Lambda = 0$  in our solution, we recover the solution obtained by Pandey and Tiwari[5].

**Particular cases:**

**Case (i):** If we consider  $\beta(t) = \frac{a}{t^2}; \alpha, a > 0$ , eqs. (24) and (25) reduce to

$$\epsilon = k_1 \alpha t^{-4} + k_1 a t^{-6}, \quad (26)$$

$$\Lambda = k_2 \alpha t^{-4} + a(k_2 + 12a)t^{-6}, \quad (27)$$

where  $k_1 = \frac{3a}{4\pi(1+\gamma)}, k_2 = \frac{12a^2\gamma}{(1+\gamma)}$ .

**Case (ii):** If we consider  $\beta(t) = \frac{a}{t}; \alpha, a > 0$ , eqs. (24) and (25) reduce to

$$\epsilon = \alpha k_3 t^{-3} + a k_3 t^{-4}, \quad (28)$$

$$\Lambda = \alpha k_3 t^{-3} + a(3a + k_4)t^{-4}, \quad (29)$$

where  $k_3 = \frac{a}{4\pi(1+\gamma)}, k_4 = \frac{2a\gamma}{(1+\gamma)}$ . It is observed from eqs. (27) and (29) that the cosmological constant  $\Lambda$ , in both cases, is a decaying function of time and it approaches a small value as time progresses (i.e., the present epoch), which explains the small value of  $\Lambda$  at present. Additionally,  $\Lambda$  also comes out positive which is consistent with the recent supernovae observations (Perlmutter et al. [33], Riess et al. [34], Garnavich et al. [35], Schmidt et al. [36]).

### 3.1 Physical properties of the model

The non-vanishing component of the flow vector,  $v^4$  is given by

$$v_4 = \frac{1}{(\alpha + \beta)}. \quad (30)$$

The reality conditions  $(\epsilon + p) > 0$  and  $(\epsilon + 3p) > 0$  lead to

$$\beta_{44} + \alpha_{11} + \frac{\alpha_1}{r} > 0, \quad (31)$$

and

$$(\alpha_1^2 - \beta_4^2) + (\alpha + \beta) \left( \beta_{44} - \frac{\alpha_1}{r} \right) > 0. \quad (32)$$

Using eq. (19) in eq. (17) gives

$$F_{14} = \frac{\left( \frac{\alpha_1}{r} - \alpha_{11} \right)^{\frac{1}{2}}}{(\alpha + \beta)^{\frac{3}{2}}}. \quad (33)$$

From eqs. (5) and (33) the current density  $\rho$  is given by

$$\rho = -\frac{(\alpha + \beta)^3}{r^2} \frac{\partial}{\partial r} \left[ \frac{r^2 \left( \frac{\alpha_1}{r} - \alpha_{11} \right)^{\frac{1}{2}}}{(\alpha + \beta)^{\frac{3}{2}}} \right]. \quad (34)$$

The non-vanishing component of the acceleration vector

$$\dot{v}_1 = v_{i;j} v^j, \quad (35)$$

is given by

$$\dot{v}_1 = -\frac{\alpha_1}{(\alpha + \beta)}. \quad (36)$$

Thus the acceleration is always directed in radial direction and the fluid flow in  $t$ -direction is uniform. If  $\alpha_1 < 0$ , acceleration is positive and if  $\alpha_1 > 0$ , there will be deceleration.

The expression for kinematical parameter expansion  $\theta$  is given by

$$\theta = 3\beta_4, \quad (37)$$

All components of rotation  $w_{ij}$  and shear  $\sigma_{ij}$  tensors are found to be zero. We observe that the expansion is time-dependent only. Hence the model (20) representing a distribution of charged perfect fluid is expanding with time but non-rotating and non-shearing.

## 4 Bulk viscous models

In this section bulk viscous models of the universe are discussed. Weinberg [16] has suggested that in order to consider the effect of bulk viscosity, the perfect fluid pressure should be replaced by effective pressure  $\bar{p}$  by

$$\bar{p} = p - \xi\theta, \quad (38)$$

where  $p$  represent equilibrium pressure,  $\xi$  is the coefficient of bulk viscosity and  $\theta$  is the expansion scalar. Here  $\xi$  is, in general, a function of time. Therefore, from eq. (21), we obtain

$$8\pi(p - \xi\theta) - \Lambda = 3(\alpha_1^2 - \beta_4^2) + (\alpha + \beta) \left( 2\beta_{44} - \alpha_{11} - \frac{3\alpha_1}{r} \right). \quad (39)$$

Thus, given  $\xi(t)$  we can solve the cosmological parameters. In most of the investigations involving bulk viscosity is assumed to be a simple power function of the energy density [21]-[23].

$$\xi(t) = \xi_0 \epsilon^n, \quad (40)$$

where  $\xi_0$  and  $n$  are constants. If  $n = 1$ , eq. (40) may correspond to a radiative fluid. However, more realistic models [24] are based on lying in the regime  $0 \leq n \leq \frac{1}{2}$ .

#### 4.1 *Model I* : ( $\xi = \xi_0$ )

In this case we assume  $n = 0$  in eq. (40) which gives  $\xi = \xi_0 = \text{constant}$ . By the use of eqs. (23) and (37) in eqs. (22) and (39), we obtain

$$4\pi(1 + \gamma)\epsilon = 12\pi\xi_0\beta_4 + (\alpha + \beta)(\alpha_{11} + \beta_{44}) \quad (41)$$

$$(1 + \gamma)\Lambda = 3(1 + \gamma)(\beta_4^2 - \alpha_1^2) + (\alpha + \beta) \left[ (1 + 3\gamma)\alpha_{11} + 3(1 + \gamma)\frac{\alpha_1}{r} - 2\beta_{44} \right] - 24\pi\xi_0\beta_4 \quad (42)$$

#### 4.2 *Model III* : ( $\xi = \xi_0\epsilon$ )

In this case we assume  $n = 1$  in eq. (40) which gives  $\xi = \xi_0\epsilon$ . By the use of eqs. (23) and (37) in eqs. (22) and (39), we obtain

$$4\pi\epsilon = \frac{(\alpha + \beta)(\alpha_{11} + \beta_{44})}{(1 + \gamma - 3\xi_0\beta_4)} \quad (43)$$

$$(1 + \gamma)\Lambda = 3(1 + \gamma)(\beta_4^2 - \alpha_1^2)$$

$$\begin{aligned}
& +(\alpha + \beta) \left[ (1 + 3\gamma)\alpha_{11} - 2\beta_{44} + 3(1 + \gamma)\frac{\alpha_1}{r} \right] \\
& - \frac{6\xi_0\beta_{44}(\alpha + \beta)(\alpha_{11} + \beta_{44})}{(1 + \gamma - 3\xi_0\beta_4)}
\end{aligned} \tag{44}$$

These eqs. (41) - (44) will supply different viable models for suitable choices of  $\beta(t)$ .

## 5 Conclusions

We have obtained conformally flat spherically symmetric cosmological models in the presence of a charged perfect fluid where the acceleration is always directed in radial direction and the fluid flow in  $t$ -direction is uniform. We have also discussed two particular cases. In both cases we observe that the energy conditions hold good and the cosmological constant is found positive and is decreasing function of time which is consistent with the present observations. The model is expanding with time but non-rotating and non-shearing.

Assuming  $\xi(t) = \xi_0\epsilon^n$ , where  $\epsilon$  is the energy density and  $n$  is the positive index, we have obtained exact solutions. The effect of the bulk viscosity is to produce a change in the perfect fluid.

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## References

- [1] H. A. Buchdahl, *Phys. Rev.* **115**, 1325 (1959).
- [2] K. P. Singh and S. R. Roy, *Proc. Natn. Inst. Sci. India*, **32A**, 223 (1966).

- [3] K. P. Singh and Abdussattar, *Gen. Rel. Grav.* **5**, 115 (1974).
- [4] S. R. Roy and Raj Bali, *Indian J. pure appl. Math.* **9**, 871 (1978).
- [5] S. N. Pandey and R. Tiwari, *Indian J. pure appl. Math.* **12**, 261 (1981).
- [6] D. R. K. Reddy, *J. Math. Phys.* **20**, 23 (1979).
- [7] V. U. M. Rao and D. R. K. Reddy, *Gen. Rel. Grav.* **14**, 1017 (1982).
- [8] K. Shanthi, *Astrophys. Space Sc.* **162**, 163 (1989).
- [9] A. Melfo and H. Rago, *Astrophys. Space Sc.* **193**, 9 (1992).
- [10] Philip D. Mannheim, *The Astrophys. J.* **391**, 429 (1992).
- [11] R. B. S. Yadav and U. Prasad, *Astrophys. Space Sc.* **203**, 37 (1993).
- [12] G. Endean, *Astrophys. J.* **479**, 40 (1997).
- [13] G. Endean, *J. Math. Phys.* **39**, 1551 (1998).
- [14] V. V. Obukhov, S. D. Odintsov and L. N. Granda, *Europhys. Lett.* **46**, 268 (1999).
- [15] M. K. Mak and T. Harko, *Int. J. Mod. Phys.* **D9**, 475 (2000).
- [16] S. Weinberg, “Gravitation and Cosmology”, J.Wiley and Sons, New York, 1972.
- [17] W. Israel and J. N. Vardalas, *Lett. nuovo Cim.* **4**, 887 (1970).
- [18] Z. Klimek, *Post. Astron.* **19**, 165 (1971).
- [19] S. Weinberge, *Astrophys. J.* **168**, 175 (1971).
- [20] L. Landau and E. M. Lifshitz, “Fluid Mechanics”, Addison-Wisley, Mass. 1962, p. 304.
- [21] D. Pavon, J. Bafaluy and D. Jou, *Class Quantum Grav.* **8**, 357 (1991);  
“Proc. Hanno Rund Conf. on Relativity and Thermodynamics”, Ed. S. D. Maharaj, University of Natal, Durban, (1996), p. 21.
- [22] R. Maartens, *Class Quantum Grav.* **12**, 1455 (1995).

- [23] W. Zimdahl, *Phys. Rev.* **D53**, 5483 (1996).
- [24] N. O. Santos, R. S. Dias and A. Banerjee, *J. Math. Phys.* **26**, 878 (1985).
- [25] T. Padmanabhan and S. M. Chitre, *Phys. Lett.* **A120**, 433 (1987).
- [26] V. B. Johri and R. Sudarshan, *Phys. Lett.* **A132**, 316 (1988).
- [27] A. Pradhan, R. V. Sarayakar and A. Beesham, *Astr. Lett. Commun.* **35**, 283 (1997).
- [28] A. Pradhan, V. K. Yadav and I. Chakrabarty, *Int. J. Mod. Phys.* **D10**, 339 (2001); *Int. J. Mod. Phys.* **D11**, 857 (2002).
- [29] I. Chakrabarty, A. Pradhan and N. N. Saste, *Int. J. Mod. Phys.* **D10**, 741 (2001).
- [30] A. Pradhan and V. K. Yadav, *Int. J. Mod. Phys.* **D11**, 857 (2002).
- [31] A. Pradhan and Aotemshi I., *Int. J. Mod. Phys.* **D11**, (2002), in press.
- [32] Ø. Grøn, *Astrophys. Space Sc.* **173**, 191 (1990).
- [33] S. Perlmutter *et al.*, *Astrophys. J.* **483**, 565 (1997), Supernova Cosmology Project Collaboration (astro-ph/9608192); *Nature* **391**, 51 (1998), Supernova Cosmology Project Collaboration (astro-ph/9712212); *Astrophys. J.* **517**, 565 (1999), Project Collaboration (astro-ph/9608192).
- [34] A. G. Riess *et al.*, *Astron. J.* **116**, 1009 (1998); Hi-Z Supernova Team Collaboration (astro-ph/9805201).
- [35] P. M. Garnavich *et al.*, *Astrophys. J.* **493**, L53 (1998a), Hi-Z Supernova Team Collaboration (astro-ph/9710123); *Astrophys. J.* **509**, 74 (1998b); Hi-Z Supernova Team Collaboration (astro-ph/9806396).
- [36] B. P. Schmidt *et al.*, *Astrophys. J.* **507**, 46 (1998), Hi-Z Supernova Team Collaboration (astro-ph/9805200).